# Formation of the cerium orthovanadate $\mathrm{CeVO}_{4}: \mathrm{DFT}+\boldsymbol{U}$ study 

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#### Abstract

We report density functional theory calculations of the structural, electronic, and thermodynamic properties of cerium orthovanadate $\left(\mathrm{CeVO}_{4}\right)$ employing the local density approximation (LDA), generalized gradient approximation (GGA-PBE), LDA $+U$, and GGA-PBE $+U$ functionals. The LDA $+U, \mathrm{GGA}-\mathrm{PBE}+U, \mathrm{LDA}$, and GGA-PBE equilibrium volumes deviate by $-2.4 \%,+3.6 \%,-7.4 \%$, and $-0.8 \%$, respectively, from experimental results. $\mathrm{DFT}+U(\mathrm{DFT})$ predicts an antiferromagnetic (ferromagnetic) insulating (metallic) ground state, which is in agreement with experimental observations. DFT $+U$ yields Ce and V ions in the III + and $\mathrm{V}+$ oxidation state, respectively. $\mathrm{CeVO}_{4}$ can be obtained by the reaction between $\mathrm{Ce}_{2} \mathrm{O}_{3}$ and $\mathrm{V}_{2} \mathrm{O}_{5}\left[\frac{1}{2} \mathrm{Ce}_{2} \mathrm{O}_{3}(\mathrm{~s})+\frac{1}{2} \mathrm{~V}_{2} \mathrm{O}_{5}(\mathrm{~s})\right.$ $\left.\rightarrow \mathrm{CeVO}_{4}(\mathrm{~s})\right]$ under an inert atmosphere, which is described as exoenergetic $\left(\left|\Delta H_{0}\right|=1.6-1.8 \mathrm{eV}\right)$ by all functionals. The reaction $\frac{1}{2} \mathrm{Ce}_{2} \mathrm{O}_{3}(\mathrm{~s})+\frac{1}{2} \mathrm{~V}_{2} \mathrm{O}_{5}(\mathrm{~s}) \rightarrow \mathrm{CeO}_{2}(\mathrm{~s})+\mathrm{VO}_{2}(\mathrm{~s})$ is exoenergetic with $\left|\Delta H_{0}\right|=0.75,0.25$, 1.70 , and 1.24 eV for $\mathrm{LDA}+U, G G A-\mathrm{PBE}+U, \mathrm{LDA}$, and GGA-PBE, respectively. Hence, $\mathrm{V}^{\mathrm{V}+}$ is more easily reduced to $\mathrm{V}^{\mathrm{IV}+}$ than $\mathrm{Ce}^{\mathrm{IV}+}$ to $\mathrm{Ce}^{\mathrm{III}+}$, but the difference is small as obtained with $\mathrm{DFT}+U$, $\mathrm{PBE}+U$, in particular. The variation of this reaction energy is due to the different performance of the various approaches for the description of the change in oxidation state of cerium, IV+ to III+ [J. L. F. Da Silva et al., Phys. Rev. B 75, $045121(2007)$ ]. The small difference between the $\mathrm{V}^{\mathrm{V}}$ and $\mathrm{Ce}^{\mathrm{IV}}$ reducibilities may have consequences for the use of $\mathrm{CeO}_{2}$ as support of $\mathrm{V}_{2} \mathrm{O}_{5}$ catalysts in selective oxidation.


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## I. INTRODUCTION

Several reactions are heterogeneously catalyzed by vanadium oxides. ${ }^{1}$ Although unsupported vanadia is an active catalyst, supported vanadium oxides are widely used. Activity, selectivity, and stability of such catalysts can be modified by varying the support. ${ }^{2-4}$ Among the different supports, e.g., $\mathrm{ZrO}_{2}, \mathrm{TiO}_{2}, \mathrm{Al}_{2} \mathrm{O}_{3}, \mathrm{SiO}_{2}, \mathrm{CeO}_{2}$, and $\mathrm{Nb}_{2} \mathrm{O}_{5}$, ceria shows a significant promoting effect. Hence, much effort has been directed toward characterizing vanadia and/or ceria catalysts (see, e.g., Refs. 5-9).

In situ spectroscopic studies of $\mathrm{V}_{2} \mathrm{O}_{5} / \mathrm{CeO}_{2}$ catalysts show clear indications that cerium orthovanadate $\left(\mathrm{CeVO}_{4}\right)$ forms upon calcination and its abundance rising in proportion to the vanadia loading on the $\mathrm{CeO}_{2}$ support. ${ }^{7-9} \mathrm{CeVO}_{4}$ is also present in used $\mathrm{V}_{2} \mathrm{O}_{5} / \mathrm{CeO}_{2}$ catalysts for oxidative dehydrogenation reactions and is associated with some loss of activity. ${ }^{7,9}$ As vanadia coverage increases, the product distribution for $\mathrm{V}_{2} \mathrm{O}_{5} / \mathrm{CeO}_{2}$ and for pure $\mathrm{CeVO}_{4}$ catalysts are quite similar. These results indicate that the same active site may be present on both catalysts, and the question of the nature of the $\mathrm{Ce}-\mathrm{O}-\mathrm{V}$ bonds in $\mathrm{CeVO}_{4}$ is particularly relevant.

The formation of $\mathrm{CeVO}_{4}$ is also of importance in the functioning of gas turbines. The burning of low-quality fuels, i.e., containing, for example, vanadium impurities, has a corrosive effect on the ceria-stabilized zirconia ceramics used as blade coatings while removing the stabilizer from zirconia and forming $\mathrm{CeVO}_{4} \cdot{ }^{10}$ In addition, $\mathrm{CeVO}_{4}$ has attracted attention due to its potential applications as a counterelectrode in electrochromic windows, ${ }^{11,12}$ gas sensors, and components of solid oxide fuel cells. ${ }^{13}$
$\mathrm{CeVO}_{4}$ can be synthesized by solid state reactions between cerium compounds and $\mathrm{V}_{2} \mathrm{O}_{5}$. The reaction between cerium oxides with Ce in the III+ oxidation state $\left(\mathrm{Ce}_{2} \mathrm{O}_{3}\right)$
and $\mathrm{V}_{2} \mathrm{O}_{5}$ is represented by $\frac{1}{2} \mathrm{Ce}_{2} \mathrm{O}_{3}(\mathrm{~s})+\frac{1}{2} \mathrm{~V}_{2} \mathrm{O}_{5}(\mathrm{~s})$ $\rightarrow \mathrm{CeVO}_{4}(\mathrm{~s})$, which requires an inert atmosphere due to the instability of the $\mathrm{Ce}^{\mathrm{III}+}$ compounds in air at high temperatures. ${ }^{14}$ Furthermore, $\mathrm{CeVO}_{4}$ can also be prepared by the conventional solid state reaction of $\mathrm{CeO}_{2}$ with Ce in the $\mathrm{IV}+$ oxidation state and $\mathrm{V}_{2} \mathrm{O}_{5}$ in air, i.e., $\mathrm{CeO}_{2}(\mathrm{~s})$ $+\frac{1}{2} \mathrm{~V}_{2} \mathrm{O}_{5}(\mathrm{~s}) \rightarrow \mathrm{CeVO}_{4}(\mathrm{~s})+\frac{1}{4} \mathrm{O}_{2}(\mathrm{~g}) .{ }^{15,16}$ In both reactions, (s) and (g) indicate solid and gas phases, respectively. Furthermore, aiming at controlling the size of the synthesized $\mathrm{CeVO}_{4}$ particles, different techniques such as the sol-gel route, ${ }^{12}$ microwave irradiation processing, ${ }^{17}$ and hydrothermal methods ${ }^{18}$ have been used.

As mentioned above, the Ce atoms are in the III+ and IV+ oxidation states in the $\mathrm{CeO}_{2}$ and $\mathrm{Ce}_{2} \mathrm{O}_{3}$ oxides, respectively. ${ }^{19}$ Because also vanadium exhibits multiple valences, the cerium oxidation state in $\mathrm{CeVO}_{4}$ is not obvious. X-ray absorption spectra indicated that the oxidation state of Ce atoms in $\mathrm{CeVO}_{4}$ is $\mathrm{Ce}^{\mathrm{III}+} .{ }^{20}$ It was further found that it was possible to create $\mathrm{Ce}^{\mathrm{IV}+}$ by adding persulfate as an oxidant. This implies that cerium is reduced during the solid state reaction of $\mathrm{CeO}_{2}$ and $\mathrm{V}_{2} \mathrm{O}_{5}$ and that vanadium remains in the $\mathrm{V}^{\mathrm{V}+}$ oxidation state. So far, $\mathrm{CeVO}_{4}$ has only been studied by means of a DV- $X_{\alpha}$ embedded cluster $\left(\mathrm{CeV}_{2} \mathrm{O}_{10}\right)$ with frozen geometry calculation. ${ }^{21}$

The determination of the electronic structure of $\mathrm{CeVO}_{4}$ appears desirable, and it can also provide useful information leading to a better understanding of ceria supported vanadia systems. Therefore, in the present work, we report a theoretical investigation of the structural, electronic, and magnetic properties of bulk $\mathrm{CeVO}_{4}$ and discuss the cerium and vanadium oxidation states. Furthermore, we present results for the $\mathrm{CeVO}_{4}$ atomization energy, heat of formation, and reaction energies for solid state reactions yielding $\mathrm{CeVO}_{4}$. As required for the evaluation of the thermodynamic properties, calculations were also performed for $\mathrm{V}_{2} \mathrm{O}_{5}$, while the total
energies of the $\mathrm{CeO}_{2}, \mathrm{Ce}_{2} \mathrm{O}_{3}$, and $\mathrm{O}_{2}$ systems were calculated using the equilibrium parameters reported in Ref. 22 (see the Appendix).

## II. THEORETICAL APPROACH AND COMPUTATIONAL DETAILS

It is known that density functional theory ${ }^{23,24}$ (DFT) within the local density approximation ${ }^{25}$ (LDA) or the generalized gradient approximation ${ }^{26}$ (GGA) often yields qualitatively incorrect results for $f$ electron systems in which the $f$ orbital overlaps are small, the $f$ bands narrow, and the $f$ electrons nearly localized. ${ }^{27}$ For example, DFT-LDA/GGA calculations yield a ferromagnetic (FM) metallic ground state for $\mathrm{Ce}_{2} \mathrm{O}_{3}$, which is in contrast with experimental observations, i.e., $\mathrm{Ce}_{2} \mathrm{O}_{3}$ is an antiferromagnetic (AFM) insulator. This particular discrepancy is due to the localized behavior of the $\mathrm{Ce} 4 f$ states $\left(\mathrm{Ce}^{\mathrm{III}+}\right)$ in $\mathrm{Ce}_{2} \mathrm{O}_{3}$, which is incorrectly described by DFT-LDA/GGA. However, DFT-LDA/GGA yields the correct ground state (nonmagnetic insulator) for $\mathrm{CeO}_{2}$ due to the delocalized nature of the $\mathrm{Ce} 4 f$ states $\left(\mathrm{Ce}^{\mathrm{IV}+}\right)$ in $\mathrm{CeO}_{2} .{ }^{22}$ The correct ground state for $\mathrm{Ce}_{2} \mathrm{O}_{3}$ has been obtained by DFT $+U$ (Refs. 22, 28, and 29) and DFT with hybrid functionals. ${ }^{22}$ There is not firm evidence for either the localized or the delocalized character of the $\mathrm{Ce} 4 f$ states in $\mathrm{CeVO}_{4}$, which is crucial to determine the oxidation state of the Ce atoms. Therefore, calculations employing both DFT and DFT $+U$ have been performed in the present work.

The implementation of the $\mathrm{DFT}+U$ method, the underlying functional (LDA or GGA), in particular, has been found to have some influence in the description of cerium compounds with $\mathrm{Ce}^{\mathrm{III}+}$ ions (see, e.g., Ref. 30). Hence, both $\mathrm{LDA}+U$ and $\mathrm{GGA}+U$ functionals are considered and discussed. $\mathrm{V}_{2} \mathrm{O}_{5}$, however, is described using plain DFT only. Mainly because $\mathrm{DFT}+U$ calculations of reaction energies for solid state reactions yielding $\mathrm{CeVO}_{4}$ requires the total energy of $\mathrm{V}_{2} \mathrm{O}_{5}$, both LDA and GGA (PBE formulation, see Ref. 26) calculations of $\mathrm{V}_{2} \mathrm{O}_{5}$ are performed.

In $\mathrm{DFT}+U$, a Hubbard $U$ term corresponding to the mean-field approximation of the on-site Coulomb interaction is added to the LDA or GGA-PBE functionals. The rotationally invariant approach proposed by Dudarev et al. ${ }^{31}$ was employed, where an effective parameter $U_{\text {eff }}=U-J$ is introduced. $U$ and $J$ are the Coulomb and exchange parameters, respectively. The Hubbard term was added only to the $\mathrm{Ce} 4 f$ states in the $\mathrm{CeVO}_{4}, \mathrm{CeO}_{2}$, and $\mathrm{Ce}_{2} \mathrm{O}_{3}$ systems, and values of 5.30 and 4.50 eV were used in the $\mathrm{LDA}+U$ and GGA-PBE $+U$ calculations, respectively. These values were calculated self-consistently by Fabris et al. ${ }^{32}$ using the linearresponse approach of Cococcioni and de Gironcoli, ${ }^{33}$ and they were employed in previous calculations on $\mathrm{CeO}_{2}$ and $\mathrm{Ce}_{2} \mathrm{O}_{3} .{ }^{22,32}$

The Kohn-Sham equations were solved using the projected augmented wave (PAW) method, ${ }^{34-36}$ as implemented in the Vienna ab initio simulation package (VASP)..$^{37,38}$ In the PAW method, the interaction between the ions and electrons are described by the standard frozen-core potentials provided with the VASP package, which were generated according to


FIG. 1. (Color online) Bulk structures of $\mathrm{CeVO}_{4}, \mathrm{CeO}_{2}, \mathrm{Ce}_{2} \mathrm{O}_{3}$, and $\mathrm{V}_{2} \mathrm{O}_{5}$. The $\mathrm{Ce}, \mathrm{V}$, and O atoms are indicated with their coordination as a superscript in parentheses. The dotted lines in the $\mathrm{V}_{2} \mathrm{O}_{5}$ structure indicate the weak bond between the $\mathrm{V}_{2} \mathrm{O}_{5}$ crystal layers.
the procedure outlined in Ref. 35. For Ce, V, and O atoms, the $(5 s, 5 p, 6 s, 4 f, 5 d),(3 s, 3 p, 4 s, 3 d)$, and $(2 s, 2 p)$ electrons were treated as valence, respectively, while the remaining electrons were kept frozen in the core. A plane-wave cutoff energy of 400 eV was chosen for all calculations, except for the stress tensor calculations for which we used a cutoff energy of 800 eV due to the slow convergence of the stress tensor with the number of basis functions. The augmentation charges were evaluated using an additional grid, which contained eight times more points than the grid for the wave functions determined from the cutoff energy. The projection operators were evaluated in reciprocal space.

The Brillouin-zone integrations were performed using Monkhorst-Pack grids, ${ }^{39}$ namely, $(6 \times 6 \times 6)$ and $(2 \times 6 \times 6)$ for $\mathrm{CeVO}_{4}$ and $\mathrm{V}_{2} \mathrm{O}_{5}$, respectively. The equilibrium volumes $V_{0}$ were calculated by minimizing the stress tensor and all internal degrees of freedom. The bulk moduli $B_{0}$ were obtained by fitting the calculated total ground-state energies for 13 relaxed structures with fixed volumes to Murnaghan's equation of state. ${ }^{40}$ All atom positions were relaxed until the forces were smaller than $0.01 \mathrm{eV} / \AA$.

## III. RESULTS AND DISCUSSION

## A. Equilibrium volumes and bulk modulus

$\mathrm{CeVO}_{4}$ has two polymorphs. The stable phase has a zircon-type body-centered-tetragonal structure with space group $D_{4 h}^{19}-I 4_{1} / a m d,{ }^{12,41}$ whereas the metastable one (stable above $400^{\circ} \mathrm{C}$ ) has a monoclinic huttonite-type structure $\left(C_{2 h}^{5}-P 2_{1} / n\right) .{ }^{15}$ We consider only the zircon-type structure. The conventional zircon-type unit cell, as shown in Fig. 1, has 4 f.u., whereas the primitive unit cell used in the actual calculations has 2 f.u. The Ce and V atoms are located at $(0,3 / 4,1 / 8)$ and $(0,3 / 4,5 / 8)$ on the $4 a$ and $4 b$ Wyckoff sites, respectively. The O atoms occupy the $16 h$ Wyckoff

TABLE I. Bulk properties of $\mathrm{CeVO}_{4}$. Equilibrium lattice constants, $a_{0}$ and $c_{0}$ (in $\AA$ ), oxygen internal parameters ( $u_{\mathrm{O}}$ and $v_{\mathrm{O}}$ ), nearestneighbor distances (in $\AA$ ), bulk modulus, $B_{0}$ (in Mbar), and total magnetic moment in $\mu_{\mathrm{B}}$ per primitive unit cell, $m$ (per cerium atom in parentheses). FM and AF indicate ferromagnetic and antiferromagnetic spin configurations, respectively.

|  | Spin | $a_{0}$ | $c_{0}$ | $u_{\mathrm{O}}$ | $v_{\mathrm{O}}$ | Ce-O | Ce-O | V-O | $B_{0}$ | $m$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LDA $+U$ | FM | 7.35 | 6.43 | 0.0708 | 0.2053 | 2.41 | 2.50 | 1.71 |  | $2.00(0.96)$ |
| LDA $+U$ | AF | 7.35 | 6.43 | 0.0708 | 0.2053 | 2.41 | 2.50 | 1.71 | 1.20 | $0.00(0.96)$ |
| GGA-PBE $+U$ | FM | 7.51 | 6.54 | 0.0720 | 0.2066 | 2.48 | 2.55 | 1.73 | $2.00(0.96)$ |  |
| GGA-PBE $+U$ | AF | 7.51 | 6.54 | 0.0721 | 0.2065 | 2.48 | 2.55 | 1.73 | 0.92 | $0.00(0.96)$ |
| LDA | FM | 7.20 | 6.36 | 0.0668 | 0.2008 | 2.33 | 2.46 | 1.72 | 1.26 | $2.00(0.70)$ |
| LDA | AF | 7.20 | 6.37 | 0.0664 | 0.2010 | 2.33 | 2.46 | 1.72 |  | $0.00(0.43)$ |
| GGA-PBE | FM | 7.37 | 6.50 | 0.0687 | 0.2025 | 2.40 | 2.51 | 1.74 | 0.90 | $2.00(0.76)$ |
| GGA-PBE $^{\text {Expt. }}{ }^{\text {a }}$ | AF | 7.36 | 6.49 | 0.0682 | 0.2029 | 2.40 | 2.52 | 1.74 | $0.00(0.65)$ |  |
| Expt. $^{\text {b,c }}$ |  | 7.35 | 6.49 | 0.0716 | 0.2067 | 2.43 | 2.52 | 1.71 |  |  |

${ }^{\text {a }}$ Reference 41 ; x-ray diffraction (XRD) of $\mathrm{Ce}\left(\mathrm{V}_{0.92} \mathrm{As}_{0.08}\right) \mathrm{O}_{4}$.
${ }^{\mathrm{b}}$ Reference 16; XRD of polycrystalline $\mathrm{CeVO}_{4}$.
${ }^{c}$ Reference 13; XRD of single-phase powders of $\mathrm{CeVO}_{4}$.
sites $\left(0, u_{\mathrm{O}}, v_{\mathrm{O}}\right)$, where $u_{\mathrm{O}}$ and $v_{\mathrm{O}}$ are internal parameters. Thus, four parameters $\left(a_{0}, b_{0}, u_{\mathrm{O}}\right.$, and $\left.v_{\mathrm{O}}\right)$ determine the complete $\mathrm{CeVO}_{4}$ structure, namely, all atomic positions in the supercell and the equilibrium volume $V_{0}$. The $\mathrm{Ce}, \mathrm{V}$, and O atoms are eight-, four-, and threefold coordinated, respectively. The eight O nearest neighbors of Ce are divided in two groups of four atoms each, between which the bond lengths differ slightly ( $0.09-0.13 \AA$ ).

The calculated structural properties are shown in Table I. The available experimental lattice parameters of $\mathrm{CeVO}_{4}$ have been determined by means of $x$-ray (powder) diffraction; ${ }^{13,16,41}$ (XRD) however the internal parameters, which determine the oxygen atomic positions, are reported only in Ref. 41. The composition of the sample used in Ref. 41 did not correspond to pure $\mathrm{CeVO}_{4}$ but to $\mathrm{Ce}\left(\mathrm{V}_{0.92} \mathrm{As}_{0.08}\right) \mathrm{O}_{4}$.

The equilibrium volume of $\mathrm{CeVO}_{4}$, which corresponds to the AFM compound with $\mathrm{LDA}+U$ and GGA-PBE $+U$ and to the ferromagnetic (FM) one with LDA and GGA-PBE, deviates by $-2.4 \%(\mathrm{LDA}+U),+3.6 \%(\mathrm{GGA}-\mathrm{PBE}+U),-7.4 \%$ (LDA), and $-0.8 \%$ (GGA-PBE) with respect to most recent experiments. ${ }^{13,16}$ The LDA value is far from being satisfactory, and that obtained using GGA-PBE does not reflect the overestimation expected for GGA functionals. ${ }^{42,43}$ This is in line with recent results for $\mathrm{Ce}_{2} \mathrm{O}_{3}$ for which the equilibrium volume is underestimated by $8.9 \%$ (LDA) and $2.7 \%$ (GGA-PBE) ${ }^{22}$ and provides a hint that the description of the electronic structure using the LDA and GGA-PBE functionals might not be correct, as in the case of $\mathrm{Ce}_{2} \mathrm{O}_{3}$.

For $\mathrm{DFT}+U$, the equilibrium volume depends significantly on the description of the plain DFT part (LDA versus GGA-PBE). The inclusion of the Hubbard $U$ term to LDA and GGA-PBE increases the volume of $\mathrm{CeVO}_{4}$ by about $5 \%$ and $3 \%$, respectively. Hence, LDA $+U$ yields an equilibrium volume closer to the experimental result than the GGA-PBE $+U$ functional. This is again similar to the description of $\mathrm{Ce}_{2} \mathrm{O}_{3}$ with a volume increase by up to $6 \%$ with the inclusion of $U$; however, the volume of $\mathrm{CeO}_{2}$ changes
only slightly $(\approx 2 \%) .{ }^{22}$ Also, for the internal parameters $u_{\mathrm{O}}$ and $v_{\mathrm{O}}$, the LDA $+U$ results are in better agreement with the experimental values ${ }^{41}$ than the GGA-PBE $+U$ results. Thus, the LDA $+U$ yielded the best description of the lattice and internal parameters for $\mathrm{CeVO}_{4}$.

Incorporation of a Hubbard $U$ term acting only on the $\mathrm{Ce} 4 f$ states increases the $\mathrm{Ce}-\mathrm{O}$ bond lengths by up to $0.08 \AA$, but the V-O bond by $0.01 \AA$ only. This result once more resembles the description of $\mathrm{Ce}_{2} \mathrm{O}_{3}$ with an increase of the average $\mathrm{Ce}-\mathrm{O}$ bond length by $0.05 \AA$ on inclusion of $U$, which is a reflection of the localization of the $\mathrm{Ce} 4 f$ states by the $\mathrm{DFT}+U$ approach. ${ }^{22}$

Furthermore, the average experimental $\mathrm{Ce}-\mathrm{O}$ bond length of the eightfold coordinated Ce atoms in $\mathrm{CeVO}_{4}$ of $2.48 \AA$ $(4 \times 2.43$ and $4 \times 2.52 \AA)$ is closer to the value of $2.50 \AA$ in $\mathrm{Ce}_{2} \mathrm{O}_{3}$ than to the $2.34 \AA$ in $\mathrm{CeO}_{2}$. In $\mathrm{CeO}_{2}$, the Ce atoms are eightfold coordinated with a $\mathrm{Ce}-\mathrm{O}$ bond length of $2.34 \AA,{ }^{44-47}$ whereas in $\mathrm{Ce}_{2} \mathrm{O}_{3}$, the Ce atoms are sevenfold coordinated with $\mathrm{Ce}-\mathrm{O}$ bond lengths of $3 \times 2.34,1 \times 2.43$, and $3 \times 2.69 \AA .{ }^{48,49}$ The increase in Ce-O bond lengths from $\mathrm{CeO}_{2}$ to $\mathrm{Ce}_{2} \mathrm{O}_{3}$ accompanies the change in the oxidation state from $\mathrm{Ce}^{\mathrm{IV}+}$ to $\mathrm{Ce}^{\mathrm{III}+} .22$ Hence, the similarity of the average $\mathrm{Ce}-\mathrm{O}$ bond length in $\mathrm{CeVO}_{4}$ and $\mathrm{Ce}_{2} \mathrm{O}_{3}$ might be taken as indication that $\mathrm{Ce}^{\mathrm{III}+}$ is present in both compounds.

Vanadium pentoxide is a nonmagnetic layered system with a soft direction perpendicular to the $\mathrm{V}_{2} \mathrm{O}_{5}$ layers (see Fig. 1). Strong covalent bonding occur between the V and O atoms in the layers, whereas weak van der Waals (vdW) interactions exist between the layers. $\mathrm{V}_{2} \mathrm{O}_{5}$ has an orthorhombic structure with space group $D_{2 h}^{13}$ Pmmn. ${ }^{50}$ The primitive unit cell comprises 2 f.u. with four crystallographically inequivalent atoms, one vanadium and three oxygen atoms. The oxygen atoms, usually denoted $\mathrm{O}^{(1)}, \mathrm{O}^{(2)}$, and $\mathrm{O}^{(3)}$, have different coordination. The vanadyl oxygen $\mathrm{O}^{(1)}$, the bridging oxygen $\mathrm{O}^{(2)}$, and the $\mathrm{O}^{(3)}$ oxygen atoms are one-, two-, and threefold coordinated, respectively (see Fig. 1). Vanadium is coordinated to five oxygen atoms within a layer $\left(\mathrm{O}^{(1)}, \mathrm{O}^{(2)}\right.$, $\left.3 \times \mathrm{O}^{(3)}\right)$, and there is a weak coordination to the $\mathrm{O}^{(1)}$ oxygen of the layer beneath.

TABLE II. Bulk properties of $\mathrm{V}_{2} \mathrm{O}_{5}$. Equilibrium lattice constants, $a_{0}, b_{0}$, and $c_{0}$ (in $\AA$ ), nearest-neighbor distances (in $\AA$ ), bulk modulus, $B_{0}$ (in Mbar ), and V and O atom positions. The Wyckoff positions are given in brackets.

|  | $a_{0}$ | $b_{0}$ | $c_{0}$ | $\mathrm{V}-\mathrm{O}^{(1)}$ | $\mathrm{V}-\mathrm{O}^{(2)}$ | V-O ${ }^{(3)}$ | $\mathrm{V}-\mathrm{O}^{\left(3^{\prime}\right)}$ | $\mathrm{V} \cdots \mathrm{O}^{(1)}$ | $B_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| LDA | 11.70 | 3.54 | 3.93 | 1.61 | 1.77 | 1.85 | 2.03 | 2.32 | 0.70 |
| GGA-PBE | 11.56 | 3.58 | 4.86 | 1.60 | 1.79 | 1.89 | 2.04 | 3.26 | 0.08 |
| Expt. ${ }^{\text {a }}$ | 11.51 | 3.56 | 4.37 | 1.58 | 1.78 | 1.88 | 2.02 | 2.79 |  |
|  |  | $\mathrm{V}[4 f](x, y, z)$ |  | $\mathrm{O}^{(1)}[4 f](x, y, z)$ |  | $\mathrm{O}^{(2)}[2 a](x, y, z)$ |  | $\mathrm{O}^{(3)}[4 f](x, y, z)$ |  |
| LDA |  | (0.1021, 1/4, 0.8999) |  | (0.1013, 1/4, 0.4908) |  | $(1 / 4,1 / 4,-0.0113)$ |  | $(-0.0676,1 / 4,0.0032)$ |  |
| GGA-PBE |  | (0.1016, 1/4, 0.8906$)$ |  | (0.1061, 1/4, 0.5619) |  | $(1 / 4,1 / 4,-0.0009)$ |  | (-0.0683, 1/4, 0.0082) |  |
| Expt. ${ }^{\text {a }}$ |  | (0.1011, 1/4, 0.8917) |  | (0.1043, 1/4, 0.5310) |  | $(1 / 4,1 / 4,+0.0010)$ |  | (-0.0689, 1/4,0.0030) |  |

${ }^{\text {a }}$ Reference 50.

Table II shows the optimized lattice parameters, $a_{0}, b_{0}$, and $c_{0}$, and the fractional coordinates using the LDA and GGA-PBE functionals and compares them with experiment. ${ }^{50}$ The deviations of the calculated values from the observed ones are in agreement with those published previously using LDA ${ }^{51-53}$ and GGA functionals. ${ }^{52,54-57}$ LDA underestimates the $\mathrm{V}_{2} \mathrm{O}_{5}$ equilibrium volume and $c_{0}$ by $9.1 \%$ and $10.1 \%$, respectively, whereas GGA-PBE overestimates them by $12.3 \%$ and $11.2 \%$, respectively. LDA and GGAPBE yield also very different values for the (weak) bond between the V atoms and the $\mathrm{O}^{(1)}$ oxygen of the layer beneath (cf. Table II). The LDA result is shorter than the measured value by $\sim 17 \%$, whereas the GGA-PBE is larger by the same amount. As discussed before, ${ }^{56,57}$ the LDA and GGA-PBE deviations in the equilibrium volume and $c_{0}$ lattice constant are a consequence of the poor description of the vdW interactions between the $\mathrm{V}_{2} \mathrm{O}_{5}$ layers, which are not properly taken into account in DFT within local and gradient-corrected functionals. ${ }^{58}$

In Fig. 2, we show the LDA and GGA-PBE potential energy surfaces of $\mathrm{V}_{2} \mathrm{O}_{5}$ as a function of the interlayer separation $c$ for the unrelaxed (equilibrium geometry) and relaxed internal degrees of freedom, with the $a$ and $b$ lattice parameters held fixed at their equilibrium values, $a_{0}$ and $b_{0}$, respectively. One can see that for both functionals, there is a minimum. We obtain with LDA an interaction energy between the $\mathrm{V}_{2} \mathrm{O}_{5}$ layers of 1.78 and 1.40 eV for the unrelaxed and relaxed geometries, respectively, whereas with GGAPBE, we find 0.23 and 0.21 eV , respectively.

As already mentioned, the coordination of V in $\mathrm{CeVO}_{4}$ is tetrahedral, which is a characteristic of all vanadate crystals. $\mathrm{AlVO}_{4}$, in particular, has been recently studied in order to get insight into the nature of the $\mathrm{Al}-\mathrm{O}-\mathrm{V}$ bonds as vanadia and/or alumina catalysts possess such bonds. ${ }^{60} \mathrm{AlVO}_{4}$ and $\mathrm{CeVO}_{4}$ are not isostructural. The unit cell of the former is triclinic with space group $P \overline{1} .{ }^{61}$ The system contains twoand threefold coordinated O atoms, five- and sixfold coordinated Al atoms, and fourfold V atoms. The $\mathrm{AlVO}_{4}$ structure possesses three symmetry-inequivalent $\mathrm{VO}_{4}$ tetrahedra. The experimental V-O bond length in $\mathrm{CeVO}_{4}$ of $1.71 \AA$ lies in the $1.62-1.84 \AA$ range of the values reported for $\mathrm{AlVO}_{4}$ with average values of $1.72,1.73$, and $1.74 \AA$ for the three $\mathrm{VO}_{4}$ tetrahedra. ${ }^{61}$

The calculated bulk modulus of $\mathrm{CeVO}_{4}$ is 1.26 and 0.90 Mbar with LDA and GGA-PBE, respectively. Inclusion of the Hubbard $U$ term yields changes of approximately $-5 \%$ and $+2 \%$, respectively. A similar change has been found for $\mathrm{Ce}_{2} \mathrm{O}_{3}$ in going from LDA to LDA $+U$, whereas GGA-PBE $+U$ yields a $3 \%$ smaller value than GGA-PBE. ${ }^{22}$ For $\mathrm{V}_{2} \mathrm{O}_{5}$, we find the usual combination of overestimated (underestimated) equilibrium volumes and underestimated (overestimated) bulk moduli, typical of GGA-PBE (LDA) (see, e.g., Ref. 43). These correlations are certainly not to be expected for $\mathrm{CeVO}_{4}$ (and $\mathrm{Ce}_{2} \mathrm{O}_{3}$ ) in line with the underestimated equilibrium volume of $\mathrm{CeVO}_{4}$ ( and $\mathrm{Ce}_{2} \mathrm{O}_{3}$ ) by both functionals. The large difference of about 0.6 Mbar between the GGAPBE and LDA bulk modulus for $\mathrm{V}_{2} \mathrm{O}_{5}$ is related to the strong (LDA) and weak (GGA-PBE) binding between the crystal layers (cf. Table II). The corresponding difference for $\mathrm{CeO}_{2}$ is about 0.3 Mbar ( 2.01 and 1.72 Mbar for LDA and GGAPBE, respectively). ${ }^{22}$


FIG. 2. (Color online) Unrelaxed and relaxed potential energy surfaces of $\mathrm{V}_{2} \mathrm{O}_{5}$ as a function of the interlayer separation between the $\mathrm{V}_{2} \mathrm{O}_{5}$ layers. $a_{0}$ and $b_{0}$ were kept fixed at their equilibrium values.


FIG. 3. (Color online) $\mathrm{CeVO}_{4}$ total and local densities of states, TDOS and LDOS, respectively; just the main $\mathrm{Ce} f, \mathrm{~V} d$, and $\mathrm{O} p$ components of the TDOS are shown. The zero energy corresponds to the top of the valence band. Only the spin-up component of the FM (AF) spin solution for LDA and GGA-PBE (LDA $+U$ and GGA-PBE $+U$ ) is shown.

## B. Magnetic and electronic properties

The total and local densities of states (DOSs) for $\mathrm{CeVO}_{4}$ and $\mathrm{V}_{2} \mathrm{O}_{5}$ are summarized in Figs. 3 and 4, respectively. $\mathrm{DFT}+U$ yields a magnetic insulating ground state for $\mathrm{CeVO}_{4}$, which is in agreement with experimental studies that characterized $\mathrm{CeVO}_{4}$ as a p-type semiconductor by the Seebeck effect. ${ }^{16}$ However, the LDA and GGA-PBE functionals incorrectly predict a magnetic metallic ground state with the Fermi level intersecting the $4 f$ band. Therefore, as shown by the DOS, the inclusion of the Hubbard $U$ term in the LDA and GGA-PBE functionals yields a narrow (occupied) $4 f$ band located about 1.0 and 1.3 eV , respectively, below the bottom of the conduction band, which is formed predominantly by $\mathrm{V} 3 d$ states and $\mathrm{Ce} 5 d+4 f$ states.

DFT $+U$ leads to AFM solutions that are lower in energy by $2 \mathrm{meV}(\mathrm{LDA}+U)$ and $1 \mathrm{meV}(\mathrm{GGA}-\mathrm{PBE}+U)$ than the FM ones. As for the $\mathrm{Ce}_{2} \mathrm{O}_{3}$ system, ${ }^{22}$ the AFM and FM magnetic states are nearly degenerated in energy. The LDA and GGA-PBE functionals predict a FM metallic ground state, which is 61 meV (LDA) and 64 meV (GGA-PBE) lower in energy than the AFM solution. The occupation of the Ce $4 f$ states is almost one electron per Ce atom, which gives rise to a spin magnetic moment of $\approx 1.0 \mu_{\mathrm{B}}$ per Ce atom. This result is consistent with the measurement of the inverse susceptibility versus temperature of a slightly reduced sample ( $\mathrm{CeVO}_{3.968}$ ), which exhibits an AFM interaction and an effective magnetic moment of $\mu_{\text {eff }}=1.29 \mu_{\mathrm{B}}$ per Ce atom. ${ }^{15}$ For neither AFM nor FM solutions from the DFT $+U$ functionals, we find any contribution from the $\mathrm{V} 3 d$ states to the total magnetic moment. In contrast, LDA and GGA-PBE


FIG. 4. (Color online) $\mathrm{V}_{2} \mathrm{O}_{5}$ total and local densities of states, TDOS and LDOS, respectively; just the main $\mathrm{V} d$ and $\mathrm{O}^{(1)}, \mathrm{O}^{(2)}$, and $\mathrm{O}^{(3)} p$ components of the TDOS are shown. The zero energy corresponds to the top of the valence band.
predict that both the $\mathrm{Ce} 4 f$ states and $\mathrm{V} 3 d$ states contribute with almost $70 \%$ and $30 \%$, respectively, to the total magnetic moment of $1.0 \mu_{\mathrm{B}}$ per Ce atom of the FM ground states (see Table I), i.e., the spin-polarized electrons are delocalized over the cerium and vanadium atoms.

As already mentioned, $\mathrm{CeVO}_{4}$ forms upon calcination of $\mathrm{V}_{2} \mathrm{O}_{5} / \mathrm{CeO}_{2}$ catalysts or during hydrocarbon oxidation reaction, and x-ray photoelectron spectroscopy ${ }^{8}$ and electron paramagnetic resonance measurements ${ }^{9}$ hinted the presence of $\mathrm{Ce}^{\mathrm{III}+}$ and $\mathrm{V}^{\mathrm{V}+}$, respectively. The present $\mathrm{DFT}+U$ results for $\mathrm{CeVO}_{4}$ support this assignment. The narrowness and occupation of the Ce $4 f$ states constitute a clear proof for the localized character of these states with concomitant existence of $\mathrm{Ce}^{\mathrm{III}+}$ ions.

For $\mathrm{V}_{2} \mathrm{O}_{5}$, LDA and GGA-PBE lead to a nonmagnetic insulating ground state (see Fig. 4), which is in agreement with previous theoretical studies using local and gradientcorrected functionals, ${ }^{51-53}$ as well as with experiment results. ${ }^{62,63}$ The calculated band gaps are 1.38 eV (LDA) and 2.06 eV (GGA-PBE). Experimentally, an optical band gap of $\sim 2.3 \mathrm{eV}$ is observed. ${ }^{62,63} \mathrm{We}$ obtain the expected underestimation of band gaps using plain DFT. However, depending on the functional, there are significant differences in the size of the band gap and the width of the valence band of predominantly $\mathrm{O} 2 p$ character (see Fig. 4). We ascribe these differences to the large discrepancy between the GGA-PBE and LDA results for the equilibrium volume, the interlayer spacing between the $\mathrm{V}_{2} \mathrm{O}_{5}$ layers (cf. $c_{0}^{\mathrm{GGA}-\mathrm{PBE}}-c_{0}^{\mathrm{LDA}}$ $=0.93 \AA$ ), in particular. The shorter interlayer spacing yields a larger bandwidth and a smaller band gap. LDA and GGAPBE calculations performed for the experimental equilibrium volumes yield band gaps of 2.21 and 2.27 eV , respectively, and similar valence bandwidths of $\sim 5 \mathrm{eV}$. The latter matches
well with values obtained from angle-integrated ${ }^{64}$ and angleresolved ultraviolet photoemission spectroscopy for crystalline $\mathrm{V}_{2} \mathrm{O}_{5}{ }^{65}$ A closer look at the local density of states (LDOS) of the oxygen atoms labeled by $\mathrm{O}^{(1)}, \mathrm{O}^{(2)}$, and $\mathrm{O}^{(3)}$ reveals differences, which reflect their different coordination (cf. Fig. 1), as earlier noticed. ${ }^{51-53}$

## C. Thermodynamic properties

The following energy differences will be discussed: (i) atomization energies $E_{\text {at }}$, (ii) heats of formation $\Delta H_{f}$, and (iii) reaction energies $\Delta H_{0}$ involved in the formation of $\mathrm{CeVO}_{4}$. To calculate $E_{\mathrm{at}}, \Delta H_{f}$, and $\Delta H_{0}$ and for the discussion below, the total energies $E_{\text {tot }}$ of $\mathrm{CeO}_{2}, \mathrm{Ce}_{2} \mathrm{O}_{3}, \mathrm{~V}_{2} \mathrm{O}_{5}$, $\mathrm{VO}_{2}$, bulk Ce , bulk V , and $\mathrm{O}_{2}$ and the $\mathrm{Ce}, \mathrm{V}$, and O atoms are required (see the Appendix).

The total energies of the free $\mathrm{Ce}, \mathrm{V}$, and O atoms were obtained from spin-polarized calculations using an orthorhombic box $\left(12 \times 13 \times 14 \AA^{3}\right)$ with no constraint on the occupation of the electronic states. The total energies of $\mathrm{CeO}_{2}$, $\mathrm{Ce}_{2} \mathrm{O}_{3}, \mathrm{VO}_{2}$, bulk Ce , bulk V , and $\mathrm{O}_{2}$ were calculated employing the equilibrium parameters reported in the Appendix (Table V), which were obtained using the same procedure and computational parameters as those used for $\mathrm{CeVO}_{4}$ in the present work. The equilibrium parameters reported in Table V for $\mathrm{CeO}_{2}, \mathrm{Ce}_{2} \mathrm{O}_{3}$, bulk Ce , and $\mathrm{O}_{2}$ are discussed elsewhere, ${ }^{22}$ whereas the structural properties of $\mathrm{VO}_{2}$ and bulk V have been discussed early in the literature and will not be discussed in this work. For the particular case of $\mathrm{O}_{2}$, the atomization energy was computed using also a hard O PAW potential with a cutoff energy of 1000 eV because of the small bond distance in $\mathrm{O}_{2}$. In the following, when the $\mathrm{O}_{2}$ atomization energy is used for the calculation of other energy differences, the value obtained with the hard PAW O potential is to be understood. Zero-point vibrational energy contributions and temperature effects were not included in our calculations.

## 1. Atomization energies and heats of formation

The Hubbard $U$ term was added only to the $\mathrm{Ce} 4 f$ states in $\mathrm{CeVO}_{4}, \mathrm{CeO}_{2}$, and $\mathrm{Ce}_{2} \mathrm{O}_{3}$, i.e., it was not added to the calculations of the free Ce atoms. Hence, the discussion of the atomization energies and heats of formation is based only on LDA and GGA-PBE results. For example, for $\mathrm{CeVO}_{4}$, the atomization energy is given by the following equation:

$$
\begin{equation*}
E_{\mathrm{at}}^{\mathrm{CeVO}_{4}}=E_{\mathrm{tot}}^{\mathrm{Ce}}+E_{\mathrm{tot}}^{\mathrm{V}}+4 E_{\mathrm{tot}}^{\mathrm{O}}-E_{\mathrm{tot}}^{\mathrm{CeVO}_{4}} \tag{1}
\end{equation*}
$$

where $E_{\mathrm{tot}}^{\mathrm{Ce} / \mathrm{V} / \mathrm{O}}$ is the total energies of the free $\mathrm{Ce}, \mathrm{V}$, and O atoms, respectively, while $E_{\text {tot }}^{\mathrm{CeVO}}{ }_{4}$ is the total energy of the bulk $\mathrm{CeVO}_{4}$. The atomization energies of $\mathrm{CeVO}_{4}$ and $\mathrm{V}_{2} \mathrm{O}_{5}$ are summarized in Table III. The heats of formation and reaction energies, which will be discussed below, can be calculated using atomization energies; hence, we report the calculated $E_{\text {at }}$ values for all relevant systems in Table III.

For all systems, the LDA atomization energies are larger than the GGA-PBE results, which have been commonly obtained in DFT studies (see, e.g., Refs. 43 and 66). To the best of our knowledge, experimental $E_{\text {at }}$ values for $\mathrm{CeVO}_{4}$ and $\mathrm{V}_{2} \mathrm{O}_{5}$ are not available. For bulk Ce , the GGA-PBE and

TABLE III. Atomization energies $E_{\mathrm{at}}$ and heats of formation $\Delta H_{f}$. All results are given per f.u.

|  | $E_{\mathrm{at}}(\mathrm{eV})$ |  |  | $\Delta H_{f}(\mathrm{eV})$ |  |
| :--- | :---: | :---: | :--- | :--- | :---: |
| System | LDA | GGA-PBE |  | LDA | GGA-PBE |
| $\mathrm{CeVO}_{4}$ | 46.93 | 40.84 |  | -19.61 | -17.80 |
| $\mathrm{~V}_{2} \mathrm{O}_{5}$ | 50.29 | 43.30 |  | -18.00 | -15.70 |
| $\mathrm{CeO}_{2}$ | 24.55 | 21.15 |  | -11.49 | -10.36 |
| $\mathrm{Ce}_{2} \mathrm{O}_{3}$ | 40.41 | 34.96 |  | -18.07 | -16.49 |
| $\mathrm{VO}_{2}$ | 22.50 | 19.22 |  | -8.24 | -6.98 |
| Bulk Ce | 5.51 | 4.58 |  |  |  |
| $\mathrm{Bulk} \mathrm{V}^{2}$ | 6.71 | 6.04 |  |  |  |
| $\mathrm{O}_{2}$ | 7.51 | 6.11 |  |  |  |
| $\mathrm{O}_{2}^{h}$ | 7.55 | 6.21 |  |  |  |

LDA $E_{\text {at }}$ values deviate by 0.26 and $1.19 \mathrm{eV} / \mathrm{Ce}$, respectively, from the experimental value of $4.32 \mathrm{eV} / \mathrm{Ce} .{ }^{67}$ For $\mathrm{O}_{2}$, similar deviations from the experimental value $\left(5.25 \mathrm{eV} / \mathrm{O}_{2}\right.$, Ref. 68) are obtained, namely, 0.96 (GGA-PBE) and 2.30 (LDA) eV/ $\mathrm{O}_{2}$. The $\mathrm{O}_{2}$ atomization energy increases by 100 and $40 \mathrm{meV} / \mathrm{O}_{2}$ if a harder O PAW potential is used with GGA-PBE and LDA, respectively.

The heat of formation of $\mathrm{CeVO}_{4}$, for example, is given by

$$
\begin{equation*}
\Delta H_{f}^{\mathrm{CeVO}_{4}}=E_{\mathrm{tot}}^{\mathrm{CeVO}_{4}}-E_{\mathrm{tot}}^{\mathrm{bulk}} \mathrm{Ce}-E_{\mathrm{tot}}^{\mathrm{bulk} \mathrm{~V}}-2 E_{\mathrm{tot}}^{\mathrm{O}_{2}} \tag{2}
\end{equation*}
$$

where $E_{\text {tot }}^{\text {bulk } \mathrm{Ce} / \mathrm{bulk} \mathrm{V} / \mathrm{O}_{2}}$ are the total energies of the bulk Ce , bulk V , and of the $\mathrm{O}_{2}$ molecule, respectively. $\Delta H_{f}^{\mathrm{CeVO}_{4}}$ can be calculated using the atomization energies in which $E_{\text {tot }}^{\text {bulk }}$ Ce/bulk $\mathrm{V} / \mathrm{O}_{2}$ should be replaced by $-E_{\text {at }}^{\text {bulk }}$ Ce/bulk $\mathrm{V} / \mathrm{O}_{2}$. The LDA and GGA-PBE heats of formation of $\mathrm{CeVO}_{4}$ and $\mathrm{V}_{2} \mathrm{O}_{5}$ are summarized in Table III. As a consequence of the larger overestimation of $E_{\text {at }}$ by LDA compared to GGA-PBE, the LDA heats of formation are also larger than the corresponding GGA-PBE results. For the case of $\mathrm{V}_{2} \mathrm{O}_{5}$, the GGA-PBE $(-15.70 \mathrm{eV})$ result is close to the experimental result of $-16.07 \mathrm{eV},{ }^{59}$ while the LDA result differs by $\approx 2 \mathrm{eV}$.

## 2. Reaction energies

As mentioned in the Introduction, $\mathrm{CeVO}_{4}$ can be synthesized by solid state reaction between $\mathrm{Ce}_{2} \mathrm{O}_{3}$ and $\mathrm{V}_{2} \mathrm{O}_{5}$, i.e.,

$$
\begin{equation*}
\frac{1}{2} \mathrm{Ce}_{2} \mathrm{O}_{3}(\mathrm{~s})+\frac{1}{2} \mathrm{~V}_{2} \mathrm{O}_{5}(\mathrm{~s}) \rightarrow \mathrm{CeVO}_{4}(\mathrm{~s}) \tag{3}
\end{equation*}
$$

The reaction energy for this reaction can be obtained by

$$
\Delta H_{0}^{\mathrm{Eq} \cdot(3)}=E_{\mathrm{tot}}^{\mathrm{CeVO}_{4}}-\frac{1}{2} E_{\mathrm{tot}}^{\mathrm{Ce}_{2} \mathrm{O}_{3}}-\frac{1}{2} E_{\mathrm{tot}}^{\mathrm{V}_{2} \mathrm{O}_{5}} .
$$

Another route to obtain $\mathrm{CeVO}_{4}$ involves $\mathrm{CeO}_{2}$ and $\mathrm{V}_{2} \mathrm{O}_{5}$, i.e.,

$$
\begin{equation*}
\mathrm{CeO}_{2}(\mathrm{~s})+\frac{1}{2} \mathrm{~V}_{2} \mathrm{O}_{5}(\mathrm{~s}) \rightarrow \mathrm{CeVO}_{4}(\mathrm{~s})+\frac{1}{4} \mathrm{O}_{2}(\mathrm{~g}), \tag{4}
\end{equation*}
$$

for which the reaction energy is given by

TABLE IV. Reaction energies ( $\Delta H_{0}$ in eV ).

| Equation No. | Reaction | LDA | GGA-PBE | LDA $+U$ | GGA-PBE $+U$ |
| :--- | :--- | :---: | :---: | :---: | :---: |
| $(3)$ | $\frac{1}{2} \mathrm{Ce}_{2} \mathrm{O}_{3}+\frac{1}{2} \mathrm{~V}_{2} \mathrm{O}_{5} \rightarrow \mathrm{CeVO}_{4}$ | -1.58 | -1.71 | -1.61 | -1.78 |
| $(4)$ | $\mathrm{CeO}_{2}+\frac{1}{2} \mathrm{~V}_{2} \mathrm{O}_{5} \rightarrow \mathrm{CeVO}_{4}+\frac{1}{2} \mathrm{O}_{2}$ | +0.88 | +0.41 | -0.10 | -0.66 |
| $(5)=(4)-(3)$ | $\mathrm{CeO}_{2} \rightarrow \frac{1}{2} \mathrm{Ce}_{2} \mathrm{O}_{3}+\frac{1}{2} \mathrm{O}_{2}$ | +2.46 | +2.11 | +1.51 | +1.12 |
| $(6)$ | $\frac{1}{2} \mathrm{~V}_{2} \mathrm{O}_{5} \rightarrow \mathrm{VO}_{2}+\frac{1}{4} \mathrm{O}_{2}$ | +0.76 | +0.87 |  |  |
| $(7)=(6)-(5)$ | $\frac{1}{2} \mathrm{Ce}_{2} \mathrm{O}_{3}+\frac{1}{2} \mathrm{~V}_{2} \mathrm{O}_{5} \rightarrow \mathrm{CeO}_{2}+\mathrm{VO}_{2}$ | -1.70 | -1.24 | -0.75 | -0.25 |

$$
\Delta H_{0}^{\mathrm{Eq} \cdot(4)}=E_{\mathrm{tot}}^{\mathrm{CeVO}_{4}}+\frac{1}{4} E_{\mathrm{tot}}^{\mathrm{O}_{2}}-E_{\mathrm{tot}}^{\mathrm{CeO}_{2}}-\frac{1}{2} E_{\mathrm{tot}}^{\mathrm{V}_{2} \mathrm{O}_{5}}
$$

The reaction energies are summarized in Table IV.
All functionals predict the first reaction [Eq. (3)] to be exoenergetic within a 0.2 eV energy range, which suggests a fortuitous error cancellation between the required total energies. Those of $\mathrm{CeVO}_{4}$ and $\mathrm{Ce}_{2} \mathrm{O}_{3}$ correspond to the FM metallic and AFM insulating ground state with plain DFT and $\mathrm{DFT}+U$, respectively. The second reaction [Eq. (4)], however, is described as endoenergetic with LDA and GGA-PBE and as exoenergetic with $\mathrm{DFT}+U$. Note that $\Delta H_{0}^{\mathrm{Eq.}}{ }^{(4)}$ values lie within a $\sim 1.5 \mathrm{eV}$ energy range.

The experimental reaction energies corresponding to Eqs. (3) and (4) are not known. However, the reduction energy of $\mathrm{CeO}_{2}$ to $\mathrm{Ce}_{2} \mathrm{O}_{3}$, i.e.,

$$
\begin{equation*}
\mathrm{CeO}_{2}(\mathrm{~s}) \rightarrow \frac{1}{2} \mathrm{Ce}_{2} \mathrm{O}_{3}(\mathrm{~s})+\frac{1}{4} \mathrm{O}_{2}(\mathrm{~g}), \tag{5}
\end{equation*}
$$

given by

$$
\begin{equation*}
\Delta H_{0}^{\mathrm{Eq} .5}=E_{\mathrm{tot}}^{\mathrm{Ce}_{2} \mathrm{O}_{3}}+\frac{1}{2} E_{\mathrm{tot}}^{\mathrm{O}_{2}}-2 E_{\mathrm{tot}}^{\mathrm{CeO}_{2}} \tag{5'}
\end{equation*}
$$

is (not very accurately) known (1.79-2.02 eV). ${ }^{69,70} \Delta H_{0}^{\text {Eq. } .5}$ equals $\Delta H_{0}^{\mathrm{Eq} .4}-\Delta H_{0}^{\mathrm{Eq} .3}$ (cf. Table IV). This reaction corresponds to the cerium oxidation state change from $\mathrm{Ce}^{\mathrm{IV}+}$ to $\mathrm{Ce}{ }^{\mathrm{III}+}$.

We have recently calculated $\Delta H_{0}^{\text {Eq. (5) }}$ and have found that LDA and GGA-PBE overestimates the $\Delta H_{0}^{\text {Eq. (5) }}$ reaction energy ${ }^{22}$ by about $0.10-0.44 \mathrm{eV}$ compared to the most recent experimental data $(2.02 \mathrm{eV}) .^{70} \mathrm{LDA}+U$ and GGA-PBE $+U$ underestimate $\Delta H_{0}^{\text {Eq. (5) }}$ by 0.5 eV and 0.89 eV , respectively. Thus, $\Delta H_{0}^{\mathrm{Eq} .}{ }^{(5)}$ values (also) lie within a fairly wide energy range $(\sim 1.3 \mathrm{eV})$. As argued in Ref. 22, these deviations are largely related to a different description of the binding of the $\mathrm{O}_{2}$ molecule for the LDA and GGAPBE functionals; the relatively small deviation for the GGAPBE case is most likely due to a fortunate cancellation of errors and not to a proper description of cerium oxides.

If we make use of the experimental value of 2.02 eV for $\Delta H_{0}^{\mathrm{Eq} .}{ }^{(5)}$ and estimate the reaction energy between $\mathrm{CeO}_{2}$ and $\mathrm{V}_{2} \mathrm{O}_{5}$ as $\Delta H_{0}^{\text {Eq. }{ }^{(3)}}+\Delta H_{0, \text { exp }}^{\text {Eq. (5) }}$, we obtain an endoenergetic reaction with all functionals $\left(\Delta H_{0}^{\mathrm{Eq.}}{ }^{(4)} \sim 0.24-0.44 \mathrm{eV}\right)$. These results, together with the discussion in Secs. III A and III B on the Ce and V oxidation state in $\mathrm{CeVO}_{4}$, indicate that out of the two reactions between cerium oxides and vanadium
pentoxide, only the reaction is exoenergetic for which the oxidation state of Ce (and V atoms) in the reactants and product is the same, i.e., $\frac{1}{2} \mathrm{Ce}_{2}{ }^{\mathrm{III}+} \mathrm{O}_{3}+\frac{1}{2} \mathrm{~V}_{2}{ }^{\mathrm{V}+} \mathrm{O}_{5}$ $\rightarrow \mathrm{Ce}^{\mathrm{III}+} \mathrm{V}^{\mathrm{V}+} \mathrm{O}_{4}$.

When $\mathrm{V}_{2} \mathrm{O}_{5}$ supported on $\mathrm{CeO}_{2}$ is used as a catalyst in selective oxidation reactions, it is assumed that $\mathrm{V}^{\mathrm{V}+}$ is reduced to $\mathrm{V}^{\mathrm{IV}+}$ (or $\mathrm{V}^{\mathrm{III}+}$ ). The reducibility of $\mathrm{V}_{2} \mathrm{O}_{5}$ can be described by the reaction

$$
\begin{equation*}
\frac{1}{2} \mathrm{~V}_{2} \mathrm{O}_{5}(\mathrm{~s}) \rightarrow \mathrm{VO}_{2}(\mathrm{~s})+\frac{1}{4} \mathrm{O}_{2}(\mathrm{~g}), \tag{6}
\end{equation*}
$$

with a reaction energy given by

$$
\begin{equation*}
\Delta H_{0}^{\mathrm{Eq} .(6)}=E_{\mathrm{tot}}^{\mathrm{VO}_{2}}+\frac{1}{4} E_{\mathrm{tot}}^{\mathrm{O}_{2}}-\frac{1}{2} E_{\mathrm{tot}}^{\mathrm{V}_{2} \mathrm{O}_{5}} . \tag{6'}
\end{equation*}
$$

Instead of the $\mathrm{V}^{\mathrm{V}+} \rightarrow \mathrm{V}^{\mathrm{IV}+}$ reduction, $\mathrm{Ce}^{\mathrm{IV}+}$ may be reduced to $\mathrm{Ce}^{\mathrm{III}+}$, which, as already mentioned, is described by Eq. (5). The reaction

$$
\begin{equation*}
\frac{1}{2} \mathrm{Ce}_{2} \mathrm{O}_{3}+\frac{1}{2} \mathrm{~V}_{2} \mathrm{O}_{5} \rightarrow \mathrm{CeO}_{2}+\mathrm{VO}_{2} \tag{7}
\end{equation*}
$$

with corresponding reaction energy at zero temperature given by

$$
\begin{equation*}
\Delta H_{0}^{\mathrm{Eq.}}{ }^{(7)}=E_{\mathrm{tot}}^{\mathrm{CeO}_{2}}+E_{\mathrm{tot}}^{\mathrm{VO}_{2}}-\frac{1}{2} E_{\mathrm{tot}}^{\mathrm{Ce}_{2} \mathrm{O}_{3}}-\frac{1}{2} E_{\mathrm{tot}}^{\mathrm{V}_{2} \mathrm{O}_{5}} \tag{7'}
\end{equation*}
$$

describes the relative reducibility of $\mathrm{CeO}_{2}$ and $\mathrm{V}_{2} \mathrm{O}_{5}$. $\Delta H_{0}^{\text {Eq. (7) }}$ equals $\Delta H_{0}^{\text {Eq. (6) }}-\Delta H_{0}^{\text {Eq. (5) }}$. Table IV shows that this reaction is exoenergetic with $1.70,1.24,0.75$, and 0.25 eV for LDA, GGA-PBE, LDA $+U$, and GGA-PBE $+U$, respectively. Hence, $\mathrm{V}^{\mathrm{V}+}$ is more easily reduced to $\mathrm{V}^{\mathrm{IV}+}$ than $\mathrm{Ce}^{\mathrm{IV}+}$ to $\mathrm{Ce}^{\mathrm{III}+}$, but the difference is small as obtained with $\mathrm{DFT}+U, \mathrm{GGA}-\mathrm{PBE}+U$, in particular. The variation in the magnitude of this reaction energy is due to the already mentioned different performances of the various approaches for the description of the change in oxidation state of cerium, $\mathrm{IV}+$ to III+. A small difference between the $\mathrm{V}^{\mathrm{V}+}$ and $\mathrm{Ce}^{\mathrm{IV}+}$ reducibilities such as $\sim 0.2 \mathrm{eV}$ may have consequences for the catalytic activity of vanadia supported on ceria. Therefore, the comparison of reactions (3) and (7) indicates that formation of the $\mathrm{CeVO}_{4}$ phase will strongly favor the $\mathrm{Ce}^{\mathrm{III}+} \mathrm{V}^{\mathrm{V}+}$ direction.

## IV. SUMMARY

We have investigated the structural, electronic, and thermodynamic properties of $\mathrm{CeVO}_{4}$ by DFT calculations using

TABLE V. Equilibrium parameters (in $\AA$ ).

|  | LDA |  |  | GGA-PBE |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| System | $a_{0}$ | $c_{0}$ |  | $a_{0}$ | $c_{0}$ |
| $\mathrm{CeO}_{2}$ | 5.37 |  |  | 5.47 |  |
| $\mathrm{CeO}_{2}$ | $5.40^{\mathrm{a}}$ |  |  | $5.49^{\mathrm{a}}$ |  |
| $\mathrm{Ce}_{2} \mathrm{O}_{3}$ | 3.77 | 5.88 |  | 3.83 | 6.08 |
| $\mathrm{Ce}_{2} \mathrm{O}_{3}$ | $3.86^{\mathrm{a}}$ | $5.96^{\mathrm{a}}$ |  | $3.92^{\mathrm{a}}$ | $6.18^{\mathrm{a}}$ |
| $\mathrm{VO}_{2}$ | 4.52 | 2.77 |  | 4.60 | 2.85 |
| $\mathrm{Bulk}^{2} \mathrm{Ce}$ | 4.51 |  |  | 4.73 |  |
| Bulk V | 2.93 |  |  | 3.00 |  |
| $\mathrm{O}_{2}$ | 1.22 |  |  | 1.23 |  |
| $\mathrm{O}_{2}^{h}$ | 1.21 |  | 1.22 |  |  |

${ }^{\text {a }}$ These results were calculated with $\mathrm{DFT}+U$.
the LDA, GGA-PBE, LDA $+U$, and GGA-PBE $+U$ functionals. $\mathrm{V}_{2} \mathrm{O}_{5}$, a reactant in the solid state reactions yielding $\mathrm{CeVO}_{4},{ }^{15,16}$ has also been considered in detail, whereas the other reactants, namely, $\mathrm{CeO}_{2}$ or $\mathrm{Ce}_{2} \mathrm{O}_{3}$, were calculated using the structures reported in Ref. 22.

For $\mathrm{CeVO}_{4}$, the equilibrium volume deviates by $-2.4 \%$ $(\mathrm{LDA}+U),+3.6 \% \quad(\mathrm{GGA}-\mathrm{PBE}+U),-7.4 \%$ (LDA), and $-0.8 \%$ (GGA-PBE) with respect to the most recent experiments. ${ }^{13,16} \mathrm{DFT}+U$ yielded larger equilibrium volumes than DFT, which is explained as a consequence of the localization of the $\mathrm{Ce} 4 f$ states in $\mathrm{CeVO}_{4}$ with $\mathrm{DFT}+U$. Similar trends were obtained for $\mathrm{Ce}_{2} \mathrm{O}_{3} \cdot{ }^{22}$ We found that the average $\mathrm{Ce}-\mathrm{O}$ bond length in $\mathrm{CeVO}_{4}$ and $\mathrm{Ce}_{2} \mathrm{O}_{3}$ are comparable and shorter by $\approx 0.15 \AA$ than that in $\mathrm{CeO}_{2}$. Thus, these structural similarities hint at Ce atoms having the same oxidation state in $\mathrm{CeVO}_{4}$ and $\mathrm{Ce}_{2} \mathrm{O}_{3}$, namely, $\mathrm{Ce}^{\mathrm{III}+}$.
$\mathrm{DFT}+U$ yielded an AF insulating ground state (only the Ce $4 f$ states contribute to the total magnetic moments), which is in agreement with experimental observations. ${ }^{15,16} \mathrm{By}$ contrast, LDA and GGA-PBE predict a FM metallic ground state. Moreover, the DFT $+U$ LDOS indicates that the highest occupied band is formed mainly by $\mathrm{Ce} 4 f$ states, which are strongly localized. These findings resemble those for $\mathrm{Ce}_{2} \mathrm{O}_{3}$ in Ref. 22, which provides further support to our oxidation state assignment of $\mathrm{Ce}^{\mathrm{III}+}$ in $\mathrm{CeVO}_{4}$.

The structural properties of $\mathrm{V}_{2} \mathrm{O}_{5}$ are in good agreement with experimental results ${ }^{50}$ with the exception of the interlayer distance between the $\mathrm{V}_{2} \mathrm{O}_{5}$ layers, which is largely underestimated (overestimated) by LDA (GGA-PBE). We attribute the shortcoming to the improper description of the weakly interacting $\mathrm{V}_{2} \mathrm{O}_{5}$ layers by the LDA and GGA-PBE functionals. ${ }^{58}$

The reaction of $\mathrm{Ce}_{2} \mathrm{O}_{3}$ with $\mathrm{V}_{2} \mathrm{O}_{5}$ is predicted to be exoenergetic by all functionals $\left(\left|\Delta H_{0}\right|=1.6-1.8 \mathrm{eV}\right)$. For the reaction of $\mathrm{CeO}_{2}$ with $\mathrm{V}_{2} \mathrm{O}_{5}$, however, the different functionals yield results in a wide energy range of $\sim 1.5 \mathrm{eV}$. Since the experimental energies are not known, comparison is not possible. Notably, for the $\mathrm{CeO}_{2} \rightarrow \frac{1}{2} \mathrm{Ce}_{2} \mathrm{O}_{3}+\frac{1}{4} \mathrm{O}_{2}$ that describes the $\mathrm{Ce}^{\mathrm{IV}+} \rightarrow \mathrm{Ce}^{\mathrm{III}+}$ change in cerium oxidation state, a similar performance of the various functionals has been observed with $\mathrm{LDA}+U$ yielding a better account for most properties. ${ }^{22}$

TABLE VI. Total energies in eV/f.u.

| System | LDA | GGA-PBE | LDA $+U$ | GGA-PBE $+U$ |
| :--- | :---: | :---: | :---: | :---: |
| $\mathrm{CeVO}_{4}$ | -57.4698 | -52.8259 | -56.4640 | -52.2515 |
| $\mathrm{CeO}_{2}$ | -28.9003 | -26.2761 | -26.9181 | -24.6363 |
| $\mathrm{Ce}_{2} \mathrm{O}_{3}$ | -47.6329 | -43.3319 | -45.5614 | -42.0403 |
| $\mathrm{~V}_{2} \mathrm{O}_{5}$ | -64.1551 | -58.9066 |  |  |
| $\mathrm{VO}_{2}$ | -28.6902 | -26.0832 |  |  |
| $\mathrm{Bulk} \mathrm{Ce}^{2}$ | -6.8984 | -5.9351 |  |  |
| $\mathrm{Bulk} \mathrm{V}^{-9.9380}$ | -9.1252 |  |  |  |
| $\mathrm{O}_{2}$ | -10.4750 | -9.8785 |  |  |
| $\mathrm{O}_{2}^{h}$ | -10.7213 | -10.0501 |  |  |
| O atom | -1.4821 | -1.8864 |  |  |
| $\mathrm{O}^{h}$ atom | -1.5873 | -1.9206 |  |  |
| Ce atom | -1.3872 | -1.3567 |  |  |
| V atom | -3.2275 | -3.0868 |  |  |

The DFT $+U$ calculated structural, electronic, and thermochemical properties of $\mathrm{CeVO}_{4}$ provide sound arguments in favor of the III+ and V+ oxidation states of Ce and V ions, respectively, and support the values inferred from experiment. ${ }^{8,9}$ Furthermore, we argue that the $\mathrm{CeVO}_{4}$ phase forming in $\mathrm{V}_{2} \mathrm{O}_{5} / \mathrm{CeO}_{2}$ catalysts is most likely to contain $\mathrm{Ce}^{\mathrm{III}+}$ and $\mathrm{V}^{\mathrm{V}+}$ cations. Thus, the present study provides an important tool for obtaining insight into the possible changes occurring to ceria supported vanadia catalysts during preparation and/or reaction.

We conclude that $\mathrm{DFT}+U$, in particular, $\mathrm{LDA}+U$, yields a better account than DFT for most of the $\mathrm{CeVO}_{4}$ properties. It is, however, also clear that shortcomings remain, in particular, in the description of the reaction energies, which illustrates the theoretical difficulties arising from electron localization. For ceria, it has been shown that further improvements are possible with hybrid functionals that include a percentage of the exact exchange. ${ }^{22}$

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## APPENDIX

Table V summarizes the equilibrium parameters of $\mathrm{CeO}_{2}$ (fluorite type, $F m \overline{3} m$ ), $\mathrm{Ce}_{2} \mathrm{O}_{3}$ (sesquioxide $A$ type, $P \overline{3} m 1$ ), $\mathrm{VO}_{2}$ (rutile), bulk Ce (face-centered cubic), bulk V (bodycentered cubic), and of the $\mathrm{O}_{2}$ molecule, which were employed in the total energy calculations required in Sec. III. The respective total energies are summarized in Table VI. They were obtained using standard PAW potentials provided within VASP and 400 eV for the cutoff energy, except for the $\mathrm{O}_{2}^{h}$ and $\mathrm{O}^{h}$, which were calculated using hard PAW and oxygen PAW potentials and 1000 eV for the cutoff energy.
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